Derivative Rules Highlights

By definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

OR

Alternate Definition for Tangent Line at a point (a, f(a))

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Vocabulary quirks:

- ✓ Slope of the Secant Line →Traditional Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ (Average Rate of Change)
- ✓ Slope of the Tangent Line →Use Derivative $f'(x) = ?$ (Instantaneous Rate of Change)
- ✓ Horizontal Tangent Line means "the slope equals 0!"

Algebraic Rules

- ✓ Power Rule $f(x) = x^n$ → $f'(x) = c \cdot nx^{n-1}$
- ✓ Product Rule $(f \cdot g)'(x) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$
- ✓ Quotient Rule $\left(\frac{f}{g}\right)'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$, $g(x) \neq 0$
- ✓ Chain Rule $fog(x) = f(g(x))$ → $fog'(x) = f'(g(x)) \bullet g'(x)$
- ✓ Implicit Derivatives Required to find $\frac{dy}{dx} = ?$ when x,y are on the same side of the equation and cannot be separated.

Trigonometric Rules

- ✓ Basic Trig Derivatives
- ✓ Chain Rule
  - -Raised to a Power Ex: $y = \cos^4 x$ → $y' = -4\cos^3 x \sin x$
  - -Unusual Angle Ex: $y = \sin(3x)$ → $y' = 3\cos(3x)$
  - -Combination Ex: $y = \tan^3(2x^2)$ → $y' = 12x\tan^2(2x^2)\sec^2(2x)$
- ✓ Inverse Trig
  - $y = Sin^{-1}x$ → $y' = \frac{1}{\sqrt{1-x^2}}$
  - $y = Tan^{-1}x$ → $y' = \frac{1}{1+x^2}$

Exponential and Logarithmic Rules

- ✓ Exponential

$$y = [a^{f(x)}] \rightarrow y' = a^{f(x)} \ln a \bullet f'(x)$$

$$y = [e^{f(x)}] \rightarrow y' = e^{f(x)} \bullet f'(x)$$

- ✓ Logarithmic

$$y = \log_a f(x) \rightarrow y' = \frac{f'(x)}{f(x)\ln a}$$

$$y = \ln f(x) \rightarrow y' = \frac{f'(x)}{f(x)}$$

Higher Order Derivatives

Notations for Derivatives of $y = f(x)$

Derivative	f' Notation	y' Notation	D Notation	Leibniz Notation
First	$f'(x)$	y'	$D_x y$	$\frac{dy}{dx}$
Second	$f''(x)$	y''	$D_x^2 y$	$\frac{d^2y}{dx^2}$
Third	$f'''(x)$	y'''	$D_x^3 y$	$\frac{d^3y}{dx^3}$
Fourth	$f^{(4)}(x)$	$y^{(4)}$	$D_x^4 y$	$\frac{d^4y}{dx^4}$
Fifth	$f^{(5)}(x)$	$y^{(5)}$	$D_x^5 y$	$\frac{d^5y}{dx^5}$
Sixth	$f^{(6)}(x)$	$y^{(6)}$	$D_x^6 y$	$\frac{d^6y}{dx^6}$
⋮	⋮	⋮	⋮	⋮
nth	$f^{(n)}(x)$	$y^{(n)}$	$D_x^n y$	$\frac{d^ny}{dx^n}$